

REPORT DOCUMENTATION PAGE		Form Approved OMB NO. 0704-0188	
Public Reporting Burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comment regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA, 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington DC 20503			
1. AGENCY USE ONLY (Leave Blank)		2. REPORT DATE:	3. REPORT TYPE AND DATES COVERED Final Report 1-Aug-2001 - 31-Jul-2006
4. TITLE AND SUBTITLE Realization of an Ultrasensitive Heisenberg-Limited Interferometer		5. FUNDING NUMBERS DAAD19-01-1-0721	
6. AUTHORS Olivier Pfister		8. PERFORMING ORGANIZATION REPORT NUMBER	
7. PERFORMING ORGANIZATION NAMES AND ADDRESSES University of Virginia Office of Sponsored Programs 1001 N. Emmett St. P.O. Box 400195 Charlottesville, VA 22904 -4195			
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) U.S. Army Research Office P.O. Box 12211 Research Triangle Park, NC 27709-2211		10. SPONSORING / MONITORING AGENCY REPORT NUMBER 42472-PH.4	
11. SUPPLEMENTARY NOTES The views, opinions and/or findings contained in this report are those of the author(s) and should not contrued as an official Department of the Army position, policy or decision, unless so designated by other documentation.			
12. DISTRIBUTION AVAILIBILITY STATEMENT Approved for Public Release; Distribution Unlimited		12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words) The abstract is below since many authors do not follow the 200 word limit			
14. SUBJECT TERMS quantum optics, nonlinear optics, squeezed states, Heisenberg-limited interferometry		15. NUMBER OF PAGES Unknown due to possible attachments	
		16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION ON THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT UL

Report Title

Realization of an Ultrasensitive Heisenberg-Limited Interferometer: Final Report

ABSTRACT

The goal of the project "Realization of an Ultrasensitive Heisenberg-Limited Interferometer," supported by ARO grant DAAD190110721 from August 1, 2001 to July 31, 2006, was the investigation of quantum interferometry with bright nonclassical light beams emitted by an ultrastable optical parametric oscillator (OPO). Theoretical studies of the Holland-Burnett Bayesian detection scheme were conducted for realistic experimental implementation in photonic quantum optics. The main result, applicable to any boson wave (eg matter waves), is that the ultimate Heisenberg limit $1/N$ (N being the average number of photons detected in the measurement) can still be reached in the presence of losses for Bayesian detection, if the losses do not exceed $1/N$. The main experimental results were the first demonstration of macroscopic Hong-Ou-Mandel quantum interference at a beam splitter and the demonstration of heterodyne polarimetry with a noise floor 4.8 dB below the interferometric shot noise limit ($1/\sqrt{N}$). The latter can be applied to enhancing the sensitivity of chiral molecule detection. Realistic extensions of this study are larger amounts of squeezing (-10 dB and beyond) as well as RF broadband phase measurements, which are a direct consequence of the stability performance of our OPO and for which we also present preliminary results.

List of papers submitted or published that acknowledge ARO support during this reporting period. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

S. Feng and O. Pfister, "Stable nondegenerate optical parametric oscillation at degenerate frequencies in Na:KTP," Journal of Optics B, Quantum and Semiclassical Optics 5, 262 (2003).

S. Feng and O. Pfister, "Quantum interference of ultrastable twin optical beams," Physical Review Letters 92, 203601 (2004). Virtual Journal of Quantum Information (June 2004).

R.C. Pooser and O. Pfister, "Particle-number scaling of the phase sensitivity in realistic Bayesian twin-mode Heisenberg-limited interferometry," Physical Review A 69, 043616 (2004).

S. Feng and O. Pfister, "Sub-shot-noise heterodyne polarimetry," Optics Letters 29, 2800 (2004).

Number of Papers published in peer-reviewed journals: 4.00

(b) Papers published in non-peer-reviewed journals or in conference proceedings (N/A for none)

Number of Papers published in non peer-reviewed journals: 0.00

(c) Presentations

Number of Presentations: 0.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts): 0

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Olivier Pfister and Sheng Feng, "Quantum interferometry with ultrastable optical parametric oscillators," (invited paper) in Proceedings of the John Hall Symposium, In Honor of John Hall on the Occasion of His 70th Birthday. World Scientific (2006).

Olivier Pfister, "Ultra-stable twin beams for quantum information and high precision measurements," (invited paper). Quantum Communication and Quantum Imaging II, SPIE Annual Meeting, Denver, August 2004.

Olivier Pfister and Sheng Feng, "Quantum interference of phase-locked twin beams," (invited paper). International Quantum Electronics Conference, OSA, San Francisco, May 2004.

O. Pfister and S. Feng, "Quantum noise of bright twin beams," in Fluctuations and Noise in Photonics and Quantum Optics II, Proc. SPIE 5468, 29 (2004).

S. Feng and O. Pfister, "Realization of an ultrastable twin-beam source for continuous-variable entanglement of bright beams," in Quantum Communications and Quantum Imaging, Proc. SPIE 5161, 109 (2004).

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts): 5

(d) Manuscripts

Number of Manuscripts: 0.00

Number of Inventions:

Graduate Students

NAME	PERCENT SUPPORTED	
Daruo Xie	0.20	No
Russell Bloomer	0.20	No
Raphael Pooser	0.20	No
(Student support counted over whole c		No
Sheng Feng	0.60	No
Gregory Jennings	0.20	No
FTE Equivalent:	1.40	
Total Number:	6	

Names of Post Doctorates

NAME	PERCENT SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT SUPPORTED	National Academy Member
Olivier Pfister	0.08	No
FTE Equivalent:	0.08	
Total Number:	1	

Names of Under Graduate students supported

NAME	PERCENT SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Personnel receiving masters degrees

<u>NAME</u>	
Gregory Jennings	No
Total Number:	1

Names of personnel receiving PHDs

<u>NAME</u>	
Sheng Feng	No
Total Number:	1

Names of other research staff

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Final Report. ARO grant DAAD190110721
Realization of an Ultrasensitive Heisenberg-Limited
Interferometer

Olivier Pfister

*Department of Physics, University of Virginia,
382 McCormick Road, Charlottesville, VA 22904-4714,
USA (434) 924 7956 opfister@virginia.edu*

Abstract

The goal of this project was the investigation of quantum interferometry with bright nonclassical light beams emitted by an ultrastable optical parametric oscillator (OPO). Theoretical studies of the Holland-Burnett Bayesian detection scheme were conducted for realistic experimental implementation in photonic quantum optics. The main result, applicable to any boson wave (eg matter waves), is that the ultimate Heisenberg limit $1/N$ (N being the average number of photons detected in the measurement) can still be reached in the presence of losses for Bayesian detection, if the losses do not exceed $1/N$. The main experimental results were the first demonstration of macroscopic Hong-Ou-Mandel quantum interference at a beam splitter and the demonstration of heterodyne polarimetry with a noise floor 4.8 dB below the interferometric shot noise limit ($1/\sqrt{N}$). The latter can be applied to enhancing the sensitivity of chiral molecule detection. Realistic extensions of this study are larger amounts of squeezing (-10 dB and beyond) as well as RF broadband phase measurements, which are a direct consequence of the stability performance of our OPO and for which we also present preliminary results.

I. STATEMENT OF THE PROBLEM STUDIED

A. The fundamental noise limits of interferometry

Quantum interferometry [1] is a universal subject: it pertains to any bosonic, and even fermionic, quantum wave. The difference between these two fundamental cases is illustrated by the number-phase Heisenberg inequality (HI) [2]

$$\Delta N \Delta \phi \geq 1 \quad (1)$$

where N is the quantum particle number in a given quantum mode (i.e. in second-quantized or quantum field theory) and ϕ is the classical phase. Do note that there exists no quantum operator for ϕ , which can be viewed as related to time. Indeed, the energy-time HI

$$\Delta E \Delta t \geq \hbar \quad (2)$$

yields the number-phase HI for a single-frequency photon field of energy $E = N\hbar\omega$ and phase $\phi = \omega t$, ω being the field's angular frequency.

For fermions, the Pauli exclusion principle yields $\Delta N \leq 1$ since two fermions can never occupy the same mode. The phase uncertainty is therefore $\Delta \phi \geq 1$ rad, which simply illustrates the well-known fact that there exists no classical wave for an ensemble of fermions. Classically, fermions always look like particles (electrons, neutrons). For bosons, ΔN can be large, in fact as large as the average number $\langle N \rangle$ of particles, which yields a macroscopic classical phase defined at the Heisenberg limit $\Delta \phi = 1/\langle N \rangle$. Note, in passing, the intermediate case of the coherent state of a harmonic oscillator, which is closest to the classical definition of a field and for which $\Delta N = \sqrt{\langle N \rangle}$ and $\Delta \phi = 1/\sqrt{\langle N \rangle}$.

An interferometer can be defined as multiple coherent wave paths brought together to superposition. Detecting that superposition necessarily depends on the phase difference between the different paths. Whereas this explanation relies on the wave nature of physics and can be entirely classical, for example in the case of light, the fundamental noise in interferometry is, however, defined by the particle nature, or Yang side to the Yin wave, to borrow from Nils Bohr's complementarity formulation. The number-/phase-difference HI between two wave

paths a and b is analogous to the single-mode one

$$\Delta(N_a - N_b)\Delta(\phi_a - \phi_b) \geq 1 \quad (3)$$

and one sees that the phase-difference noise will be conditioned by the number-difference statistics, in the case of a minimum uncertainty state (equality in above equation): lowering the noise floor of the measurement of the phase difference between two arms of an interferometer will therefore require an increase in the number-difference noise. Another, more hand-waving way of stating the latter is to say that maximizing the indistinguishability between the interferometer arms maximizes the interference signal-to-noise.

B. The wave-splitting noise

In creating the aforementioned multiple coherent waves, one can start with a single initial wave, subsequently “split” into subwaves for example by a beam splitter. In classical optics, the subwaves thus obtained are coherent and can then be made to interfere. In quantum optics, such a splitting alters particle statistics and creates noise [3]. It is easy to see qualitatively by considering a Fock state containing N photons impinging on a 50% reflecting mirror (say port a in Fig.1): the statistics of photons scattering off the mirror follow a binomial

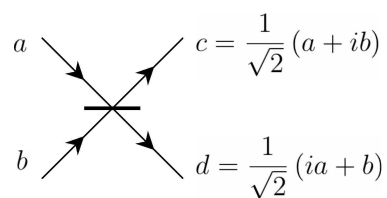


FIG. 1: *Balanced beam splitter. The input and output modes have annihilation operators (a,b) and (c,d) respectively. The reflection/transmission coefficients are $r = it = 2^{-1/2}$. The wave vectors are aligned so that $\hat{k}_{c,d} = \hat{k}_{b,a}$.*

law and the standard deviation of the photon number in each output port (reflection c and transmission d) of the beam splitter, as well as of their difference, is proportional to \sqrt{N} , despite the initial state having no fluctuations whatsoever! The rigorous quantum optical derivation gives the exact same result, which stems from the interference of the bright state $|N\rangle_a$ in input port a with the *vacuum* state $|0\rangle_b$ entering the other “unused” input port b of

Fig.1 [3]. The conventional interferometer therefore yields the shot-noise limit solely because of the splitting noise.

In order to break the beam splitter shot-noise limit, it is necessary to modify the “unused” input. This can be done by vacuum squeezing [4, 5] or by using number-correlated inputs [6, 7] which is the method that was used experimentally in this proposal. For a fairly complete review spanning many physics fields, see Ref. [1].

II. SUMMARY OF THE MOST IMPORTANT RESULTS

A. Macroscopic quantum interference of ultrastable twin optical beams [8, 9]

S. Feng and O. Pfister, J. Opt. B, Quantum Semiclass. Opt. **5**, 262 (2003).

S. Feng and O. Pfister, Phys. Rev. Lett. **92**, 203601 (2004).

This first result featured the successful stabilization of a standard nondegenerate OPO at unprecedented levels in a quantum optics experiment, which enabled the observation of quantum interference at the macroscopic level. This OPO (Fig.2) was the first one built in our group. As is well known, an OPO is based on a nonlinear crystal which downconverts pump photons into pairs of signal photons. The signals’ frequencies and wave vectors add up to those of the pump, as required by the phase-matching condition in nonlinear optics. In our case, all wave vectors were collinear and the signal beams had orthogonal polarizations, one of them parallel to the pump polarization. The nonlinear crystal was Sodium-doped Potassium Titanyl Phosphate (Na:KTiOPO₄ or Na:KTP), placed in a laser cavity constituted of two dichroic mirrors with ultra-low absorption and scattering losses. One mirror entirely transmitted the pump wavelength (532 nm in our case) and entirely reflected the signal wavelength (1064 nm for both beams). The other mirror reflected entirely at 532 nm and had a 1 % transmission for output coupling at 1064 nm. This OPO cavity was thus doubly resonant for both signal beams and double-passed the pump through the crystal. (Note that special care must be taken in designing the pump-reflecting mirror so that the phases between the three fields participating in the nonlinear interaction are still phasematched

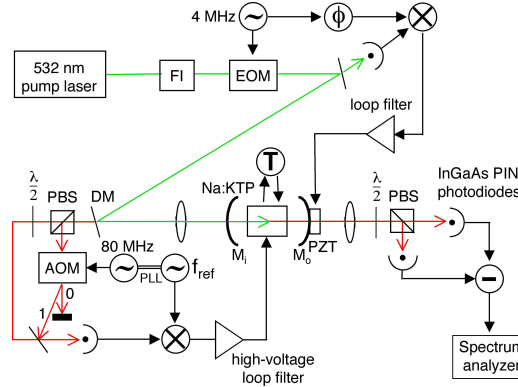


FIG. 2: *Simplified experimental setup.* Green lines denote the 532 nm pump beam. Red lines denote the 1064 nm OPO twin beams. The servo loop at the top of the figure is the optical frequency-lock loop, the one at the bottom left is the optical phase-lock loop. *T*: crystal temperature servo loop. *M_i*: input mirror (reflectivity: $\simeq 0\%$ @ 532 nm; 99.99% @ 1064nm). *M_o*: output mirror (reflectivity: 99.995% @ 532 nm; 99% @ 1064nm). *FI*: Faraday isolator. *EOM*: electro-optic modulator. *DM*: dichroic mirror. *PBS*: polarizing beam splitter. *AOM*: acousto-optic modulator. *PLL*: (electronic) phase-lock loop (the 80 MHz and f_{ref} sources are electronically phase-locked together). *PZT*: piezoelectric transducer.

when they all reenter the crystal. To achieve this, the mirror's phase shifts must correct the dispersion of the air in the roundtrip path between the mirror and the crystal.) The oscillation threshold is reached when the pump power is high enough for the spontaneous parametric downconversion into the cavity mode to equal, over a roundtrip, the mirror transmission (and other) losses. We worked with typical continuous-wave powers of 4 mW in each OPO beam, with a pump threshold power of 60 mW.

It is important to note at this point that the OPO threshold is what could be called a Rubicon (hopefully not a Styx) of quantum optics: crossing it has drastic consequences and it has not been often crossed in the vast majority of quantum optics experiments, except by the group of Claude Fabre in Paris [10] (see also bright pulsed twin beam generation in OPA's [11, 12]). Using OPO's above threshold brings in a new regime in which the system is not an *amplifier* (OPA, with or without a mode-filtering cavity) but an *oscillator*. The difference is, of course, important: an amplifier's output is largely defined by its input (be it

a vacuum field or a seed laser beam) whereas an oscillator's output depends on the operating point, which in turn is defined by the oscillator's internal dynamics that involves the cavity's feedback and can sometimes become very complex and chaotic. There is still a great deal to discover about the quantum properties of the oscillator regime, in lasers (such as the two-photon laser [13]) or in OPO's.

Above threshold, the nondegenerate OPO emitted "twin quantum beams" that were correlated at the photon level due to the photon pair emission process. The beams were orthogonally polarized and thus easily separated and detected with high-efficiency InGaAs PIN photodiodes. We obtained a 6 dB reduction of the intensity difference noise below the total shot noise of the two beams (Fig.3, center). Because of this photon number correlation, a purely quantum interference effect was then observed that is a generalization of the well known Hong-Ou-Mandel interference for pairs of indistinguishable photons impinging on the two input ports a and b of a beam splitter [14]. In the latter case, four scattering amplitudes are possible, each photon being either reflected and transmitted. For a balanced lossless beam splitter, the two amplitudes with photons both transmitted and both reflected interfere destructively, thereby leading to an output state of the form $|2\rangle_c|0\rangle_d + |0\rangle_c|2\rangle_d$ (no $|1\rangle_c|1\rangle_d$ term). Now, in our case, the input state was not $|1\rangle_a|1\rangle_b$ but had much larger photon numbers and common-mode statistics. The basic physics, however, remains the same: there is quantum interference of the amplitudes of indistinguishable output states. For the simpler twin Fock state input $|n\rangle_a|n\rangle_b$, the effect was calculated [15] using $\text{su}(2)$ algebra [16, 17]. The light state emitted by the OPO is not a twin Fock state but rather of the form $\sum_n c_n |n\rangle_a |n\rangle_b$ or, more generally, a density operator of the form $\sum_{nm} \rho_{nm} |nn\rangle \langle mm|$. Nonetheless, the quantum interference effect is still taking place, as we proved theoretically some time ago [7]. The effect of this interference was essentially to bunch all photons at either beam splitter's output, thereby creating an extremely noisy photon-number difference (ideally, $\Delta(N_c - N_d) \propto \langle N_c + N_d \rangle$ instead of $\langle N_c + N_d \rangle^{1/2}$) and therefore a squeezed phase difference (ideally, $\Delta(\phi_c - \phi_d) \propto \langle N_c + N_d \rangle^{-1}$ instead of $\langle N_c + N_d \rangle^{-1/2}$), in accordance with the number-phase Heisenberg uncertainty. This is of interest for Heisenberg-limited interferometry, as was first proposed by Holland and Burnett [6, 7]. It is also equivalent to the generation of a bright entangled state, albeit a nonmaximally entangled one. The experimental challenge lied in the stabilization of the OPO frequencies to the level of in-

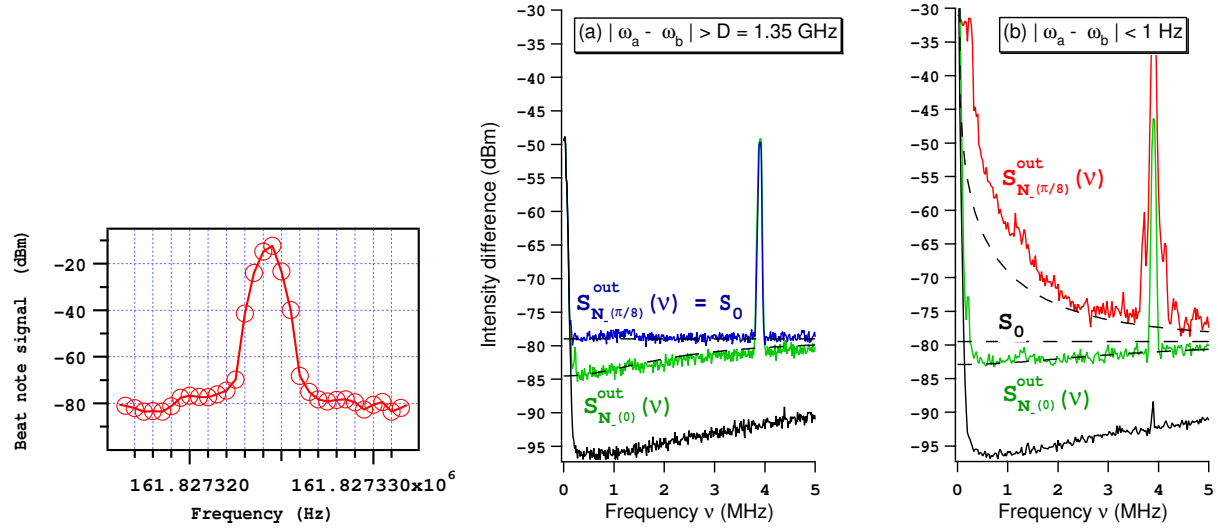


FIG. 3: *Left: interference (beat note) signal between the 2 OPO beams. The width of the oscillation peak at the difference of the optical frequencies is limited by the 1 Hz resolution bandwidth of the spectrum analyzer. Note the span of the spectrum is 16 Hz. Center and Right: power spectra of the photocurrent difference of the output of a beam splitter fed by distinguishable (Center, blue trace) and indistinguishable (Right, red trace) OPO twin beams. The green traces are the squeezed photocurrent differences before the beam splitter. Solid black line is the electronic detection noise. Dashed black lines are theoretical predictions. The 3.9 MHz peak is a modulation sideband.*

distinguishability compatible with the measurement bandwidth (30 kHz or less) required to resolve the squeezing bandwidth. We used three different electronic servo loops to control the OPO: one for the crystal's temperature (error of a few 0.1 mK), one for the OPO cavity length (10 pm error), and one for the phase difference of the OPO twin beams. As a result, the frequency difference error was *less than 1 Hz*, as displayed in the left graph of Fig.3. This ensured an exceptional degree of indistinguishability once the frequency difference was tuned to zero by temperature-tuning the OPO's Na:KTP crystal. We then turned to the demonstration of the macroscopic Hong-Ou-Mandel interference with ultrastable twin beams. The experimental results are presented in the center and right graphs of Fig.3. The blue trace is at the shot noise level, demonstrating the contamination by vacuum fluctuations of this OPO self-heterodyne signal. The red trace is the OPO self-homodyne signal and displays a large excess of quantum noise. This noise can be viewed as the signature of macroscopic

Hong-Ou-Mandel interference or, equivalently, as the signature of the noisy phase difference of the OPO beams before the beam splitter (since the beams are number-difference squeezed there, and in a minimum uncertainty state).

B. Sub-shot-noise heterodyne polarimetry [18]

S. Feng and O. Pfister, Opt. Lett. **29**, 2800 (2004).

In this application of phase-locked twin beams to Heisenberg-limited interferometry, the OPO phase-difference was locked to a 1 MHz stable radiofrequency and a minute unknown polarization rotation was exerted by the wave plate before the polarizing beam splitter in Fig.2. The resulting 1 MHz beat note was free of classical noise and sat on a quantum noise floor -4.8 dB from the shot noise level, thereby demonstrating a threefold sensitivity improvement over classical interferometry (Fig.4).

C. Particle-number scaling of the phase sensitivity in realistic Bayesian twin-mode Heisenberg-limited interferometry [19]

R.C. Pooser and O. Pfister, Phys. Rev. A **69**, 043616 (2004).

This theoretical work was based on the Bayesian detection method proposed by Holland and Burnett [6] for the homodyne twin-beam case, where no direct signal can be observed. The method consists in several dynamical measurements of independent fluctuations pertaining to the same experimental conditions (i.e. input light state and interferometer phase shift) and subsequent reconstruction of the joint probability distribution for all independent measurements by use of Bayes' theorem. We used a Monte Carlo simulation program to examine the effect of losses on this highly nonlinear detection scheme, with its experimental implementation in the back of our minds. The main result of this study was that the Bayesian detection of quantum interferometry with twin beams stays Heisenberg limited as long as the loss rate doesn't exceed $1/N$, for a $|N\rangle_1|N\rangle_b$ input. This is somewhat surprising, even though not of great use for photon-based implementations, for which the losses will always

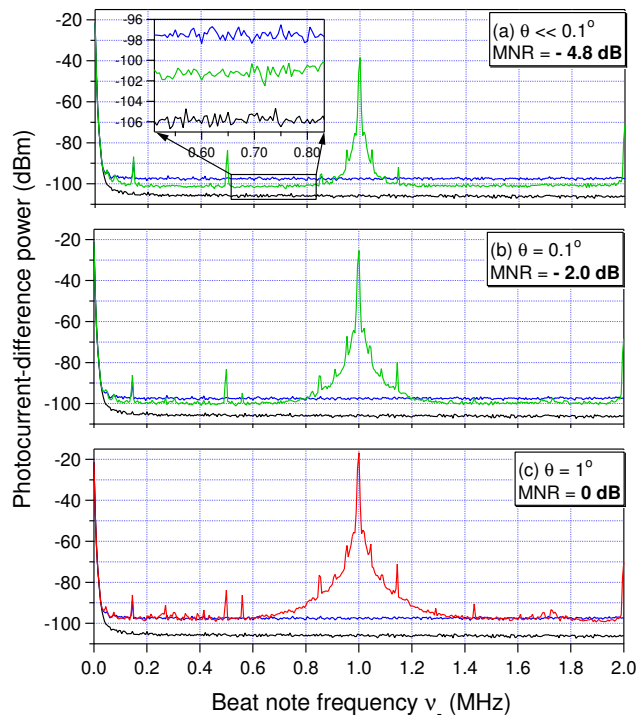


FIG. 4: *Sub-SNL heterodyne polarimetry signals. In all three figures, the two flat traces are the shot noise level (upper) and the detection electronics noise (lower). The peaked trace is the twin-beam beat note signal, the resolution and video bandwidths are 3 kHz (100 averages). The maximum beat note amplitude ($\theta = 22.5^\circ$) is 25 dBm. MNR: Measurement-noise reduction.*

be much larger. (Recall that we want N large to increase sensitivity.) For other type of interferometers, however, such as atom ones, this may prove a very interesting result for the implementation of the Holland-Burnett method.

D. Preliminary result: Broadband squeezing for RF detection [20]

D. Xie, M. Pysher, J. Jing, and O. Pfister, in preparation.

In this experiment, we exploit the outstanding stability of the OPO and of all optical paths, which allows us to acquire the *whole squeezing spectrum*, rather than a zero span scan at a single frequency, thus yielding broadband signal detection enhancement. This is applied to

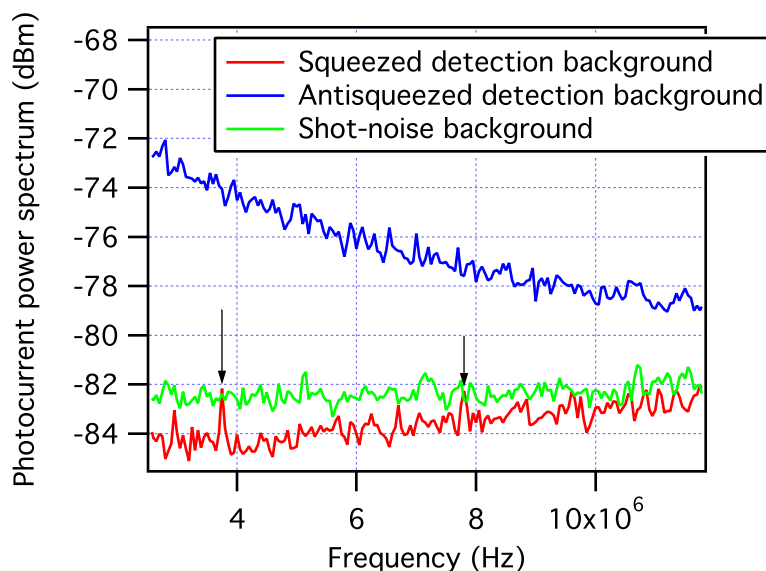


FIG. 5: Photodetection power spectrum of the homodyne beat of a phase-modulated laser with an ultrastable squeezed beam (red trace), compared with the laser's shot noise level (green trace) and the antisqueezed noise (blue trace). The stability of the OPO and of all optical path allows us to acquire the whole squeezing spectrum rather than a zero span scan at a single frequency.

the detection of two-tone phase modulation of a laser beam by an electro-optic modulator driven by millivolt oscillating signals at 3.75 and 7.80 MHz (arrows in Fig.5). The sensitivity enhancement is simply equal to the squeezing amount, a preliminary 2 dB in this case. Further studies will focus on increasing the squeezing level as well as the bandwidth. These studies will necessitate the use of novel nonlinear materials, which will be developed through collaborations.

E. Development of human resources, conclusion, and outlook

All these results have been made possible by the exceptionally motivated graduate students of the group, in particular Dr. Sheng Feng, and Raphael Pooser and Daruo Xie. Sheng Feng defended his Ph.D. thesis in May 2005 and was the first Ph.D. student to graduate from the Quantum Optics and Quantum Information group at UVa. He is now a postdoctoral Research Associate with Prof. Prem Kumar at Northwestern. Raphael Pooser and Daruo

Xie are both on pace to defend their Ph.D. in the coming year.

In conclusion, the project was very successful and further studies using novel nonlinear materials such as photonic bandgap materials will probably keep moving the performance frontier of quantum interferometry forward. It is the belief of the PI at this time that performance in excess of 10 dB squeezing, and possibly reaching 20 dB, can probably be achieved in integrated optical structures, such as waveguides, thus yielding one to two order-of-magnitude sensitivity improvement for photonic interferometry, the ultimate experimental limit being fixed by residual light absorption in the nonlinear crystal. Note that quantum interferometry with matter waves, e.g. Bose-Einstein condensates, is not necessarily subject to the same experimental limitations and could possibly reach higher level of squeezing, even though particle number is likely to be smaller. (This will require taming the mean-field interaction first, since it creates additional phase noise in the condensates.) Finally, any physical system that can be cast as a $SU(2)$ interferometer (Mach-Zehnder interferometer, Ramsey interferometer...) can use this quantum analysis. Another interesting direction, though more challenging in experimental photonic quantum optics, is $SU(1,1)$ interferometry, where the interferometer beam splitters are replaced with nonlinear media [17].

-
- [1] A. Luis and L.L. Sánchez-Soto, Progress in Optics **41**, Ed. E. Wolf, Elsevier (2000) p. 421.
 - [2] J.-M. Lévy-Leblond and F. Balibar, *Quantics. Rudiments of Quantum Physics*, North-Holland, Amsterdam, 1990.
 - [3] C.M. Caves, Phys. Rev. Lett. **45**, 75 (1980).
 - [4] C.M. Caves, Phys. Rev. D **23**, 1693 (1981).
 - [5] M. Xiao, L.-A. Wu, and H.J. Kimble, Phys. Rev. Lett. **59**, 278 (1987); P. Grangier, R.E. Slusher, B. Yurke, and A. LaPorta, Phys. Rev. Lett. **59**, 2153 (1987).
 - [6] M.J. Holland and K. Burnett, Phys. Rev. Lett. **71**, 1355 (1993).
 - [7] T. Kim, O. Pfister, M.J. Holland, J. Noh, J.L. Hall, Phys. Rev. A **57**, 4004; **58**, 2617 (1998).
 - [8] S. Feng and O. Pfister, J. Opt. B, Quantum Semiclass. Opt. **5**, 262 (2003).
 - [9] S. Feng and O. Pfister, Phys. Rev. Lett. **92**, 203601 (2004).

- [10] A. Heidmann, R.J. Horowicz, S. Reynaud, E. Giacobino, and C. Fabre, Phys. Rev. Lett. **59**, 2555 (1987).
- [11] O. Aytür and P. Kumar, Phys. Rev. Lett. **65**, 1551 (1990).
- [12] D.T. Smithey, M. Beck, M. Belsley, and M.G. Raymer, Phys. Rev. Lett. **69**, 2650 (1992).
- [13] O. Pfister, W.J. Brown, M.D. Stenner, and D.J. Gauthier, Phys. Rev. Lett. **86**, 4512 (2001).
- [14] C.K. Hong, Z.Y. Ou, and L. Mandel, Phys. Rev. Lett. **59**, 2044 (1987).
- [15] R.A. Campos, B.E.A. Saleh, M.C. Teich, Phys. Rev. A **40**, 1371 (1989).
- [16] K. Wódkiewicz and J.H. Eberly, J. Opt. Soc. Am. B **2**, 458 (1985).
- [17] B. Yurke, S.L. McCall, and J.R. Klauder, Phys. Rev. A **33**, 4033 (1986).
- [18] S. Feng and O. Pfister, Opt. Lett. **29**, 2800 (2004).
- [19] R.C. Pooser and O. Pfister, Phys. Rev. A **69**, 043616 (2004).
- [20] D. Xie, M. Pysher, J. Jing, and O. Pfister, in preparation.